A Power Analysis of Two Surveillance Methods In Terms of Average Run Lengths

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Incidence Rates: OK or Not OK

- Are rates too high? Is action needed?
- Waiting too long→ Needless death, $$ cost.
- Reacting to small increases → needless $$ costs, job loss.

Incidence and Quality Control (QC)

- Incidence rates over time act like rates/counts of defective products in a production line.

- We (logically) apply industrial QC procedures to disease surveillance.

QC Method #1: Shewhart (1931)

GIVEN: Process w/(mean, SD) = (μ, σ) (e.g., lengths of 12” rulers, measures of 8mm bolts).

GIVEN: w = mean of set of individual measurements

- Shewhart signals process ‘out-of-control’ if
  $w \notin (\mu - 3\sigma, \mu + 3\sigma)$
- Special case: c-charts (number of defects-analogous to disease surveillance)

QC Method #2: CUSUM

CUSUM = Cumulative Sum

- If the sum of a set of consecutive measurements reaches a threshold point, then CUSUM signals process out of control.
- IMPORTANT: Shewhart takes individual measurements in isolation (memoryless), CUSUM takes a series of measures.

Modified Shewhart: Supplementary Runs Rules

- Attempt to fix 'memoryless' character of Shewhart
- If set of successive points have unusual pattern→ 'out-of-control' signal.
- Example: If two out of three successive w's are out of $(\mu-2\sigma, \mu+2\sigma)$ (2 SD's from mean) → 'out-of-control' signal

Modified Shewhart for Incidence: TEXAS method (Hardy et. al 1990) (1 of 2)

- Proposed for “monitoring the health status of a community potentially exposed to a hazardous environment.”
- TEXAS observes incidence over series of time intervals
- Uses two consecutive measures to determine whether further study or action is warranted.

Modified Shewhart for Incidence: TEXAS method (Hardy et. al 1990) (2 of 2)

- “action” level (RED) reached if # cases for a time period is greater than a high preset value.
- RED level→ out-of-control signal
- “alert” level (YELLOW) reached if # cases for a time period is g.t. a lower level (but below the “action” level).
- 2 consecutive YELLOW’s → out-of-control signal.

Research Question

Statement-First Form

- *Given that the incidence rate is higher than it should be* which procedure (TEXAS or CUSUM) is more likely under various null and alternative hypotheses to indicate such? (e.g., which procedure has greater statistical power?)

Statement-Second Form

If

- Ho: Incidence rate is $\lambda = \lambda_0$

  (No need for further study)

and

- Ha: Incidence rate is $\lambda > \lambda_0$ (Further study warranted),

Which procedure will tend to reject Ho in the fewest number of time periods?

Rest of Talk

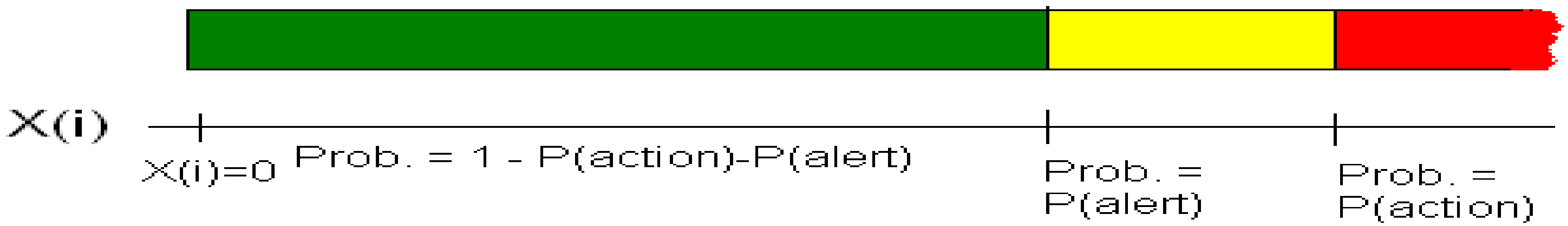
- Present forms of TEXAS and CUSUM that assume incidents rates are Poisson distributed (page 3, 4 of paper).
- Equate Type I errors for TEXAS and CUSUM (α vs. ARL-Average Run Length) (p. 4, 5 of paper)
- Simulation Procedure (p. 5,6 of paper)
- Results/Discussion (Remainder)

Hypotheses & Notation

- Acceptable incidence rate = λ_1
- Ho: $\lambda = \lambda_1$ (Rate acceptable)
- Ha: $\lambda > \lambda_1$ (Rate too high—further study warranted)
- X_i = # of cases at time I
- $X_i \sim$ Poisson(λ_1)
- We reject Ho when a signal occurs indicating that the process is out of control.

TEXAS method

Alert level (RED), action level (YELLOW) and neither for a single observation X(i) at time i.



- Type I error = Significance Level = α
- P(action) = Prob. that X_i reaches action-RED-level in *single* time interval i → process out of control at i.
- P(alert) = Prob. X_i reaches alert-YELLOW-level at time i.
- If alert level reached at i and at i + 1 → process out of control at time i + 1.

Relationship of P(alert), P(action) and α

We can show (see paper) that

$$[P(alert)]^2 = [P(action)]^2 - 2P(action) + \alpha$$

CUSUM method

- CUSUM measures shift from a rate $\lambda = \lambda_1$ to a second rate $\lambda = \lambda_2 > \lambda_1$
- CUSUM C(i) at time i is

C(i) = max(C(i-1)+X(i)-k), where

---- C(0) = 0

---- $k = (\lambda_2 - \lambda_1) / (\ln(\lambda_2) – \ln(\lambda_1))$

The process sends an out-of-control signal if C(i) > h, where h is a function of λ_1, λ_2, and an in-control Average-Run Length (ARL).

Type I Error: α vs. ARL (1 of 2)

- TEXAS: Type I Error = α
- CUSUM: Type I Error = ARL
- We must equate the two—see “The ARL vs. ALPHA PROBLEM” in the paper.

Type I Error: α vs. ARL (2 of 2)

- Problem: Poisson (DISCRETE) distribution → Discrete number of ARL's for a given null and alternative hypotheses.
- Solution: We find a set of possible ARL's and then GIVEN those ARL's we calculate α's corresponding to those ARL's (see "PROCEDURE" in paper.

Procedure

Define:

$$H_o : \lambda = \lambda_1$$

$$H_A : \lambda = \lambda_1 + 2^n \sqrt{\lambda_1}$$

where

$$\lambda_1 \in \{0.1, 0.2, 0.5, 1, 2, 3, 5, 10, 25, 50, 100\}$$

$$n \in \{-3, -2, -1, 0, 1, 2, 3\}$$

Hypothetical ARL's

$$ARL_{CUSUM\_HYPOTHETICAL} \in \left\{ \begin{array}{l} 10,\ 20,\ 30, \ldots, 100, \\ 120, 140, 160, \ldots, 300, \\ 330, 360, 390, \ldots, 600, \\ 640, 680, 720, \ldots, 1000 \end{array} \right\}$$

From ARL's to α's (1 of 2)

$$\left(\lambda_1, n, ARL_{CUSUM\_HYPOTHETICAL}\right)$$

w/ANYGETH (Hawkins and Olwell) →

(1) exact ARL_{CUSUM_ACTUAL} and

(2) corresponding RATIONAL h's and k's that are

(3) as close as possible to the hypothetical ARL's

From ARL's to α's (2 of 2)

α's for TEXAS were calculated using the expression

$$ARL_{CUSUM\_ACTUAL} = \frac{1 + p_{alert}}{p_{alert}^2 + p_{alert}p_{action} + p_{action}}$$

with

$$p_{action} = g\alpha,$$

$$g \in \{0.01, 0.02, 0.03, \ldots, 0.49, 0.4999\}$$

$$p_{alert}^2 = p_{action}^2 - 2p_{action} + \alpha$$

TEXAS vs. CUSUM: Underlying Idea

- Power of a test = Probability that the test will CORRECTLY reject a FALSE null hypothesis.
- Related to Type II error.
- The more powerful procedure will CORRECTLY reject the null hypothesis in the SHORTER number of iterations.

Simulation (1 of 3)

- Assume Type II Error Assumption
$$X_i \sim Poisson\left(\lambda_1 + 2^n \sqrt{\lambda_1}\right)$$
- 5000 CUSUM Iterations for each
$$\left(\lambda_1, n, ARL_{CUSUM\_ACTUAL}\right)$$
- 5000 TEXAS Iterations for each
$$\left(\lambda_1, n, ARL_{CUSUM\_ACTUAL}, g\right)$$
- Iterations for TEXAS and CUSUM are ordered

Simulation (2 of 3)

- For specific $\left(\lambda_1, n, ARL_{CUSUM\_ACTUAL}, g\right)$
- TEXAS more powerful than CUSUM if for at least 4500 of the 5000 ORDERED simulations (TEXAS <= CUSUM)
- CUSUM more powerful than TEXAS if for at least 4500 of the 5000 ORDERED simulations (CUSUM <= TEXAS)
- Otherwise, result is inconclusive

Simulation (3 of 3)

For $\left(\lambda_1, n, ARL_{CUSUM\_ACTUAL}\right)$ and all g:

- TEXAS more powerful than CUSUM if for at least 4500 of the 5000 ORDERED simulations [max(TEXAS for all g) <= CUSUM]
- CUSUM more powerful than TEXAS if for at least 4500 of the 5000 ORDERED simulations [CUSUM <= min(TEXAS for all g)]
- Otherwise, result is inconclusive

Results

Issues

- Focus on Minimum, maximum cases
- 280 potential cases for a given null hypothesis (7 n's, 40 hypothetical cases)
- Discrete Poisson Dist'n → Limited # of actual ARL's → Duplicate ARL's discarded

Table 1 of Paper

Table 2: By n's in $H_A : \lambda = \lambda_1 + 2^n \sqrt{\lambda_1}$

	Result		
Alternative Hypothesis	TEXAS More Powerful	CUSUM More Powerful	Results Inconclusive
n = -3	48	0	362
n = -2	33	0	379
n = -1	13	0	360
n = 0	7	0	349
n = 1	2	0	276
n = 2	7	17	129
n = 3	8	85	44

Conclusion

- TEXAS more powerful for smaller λ's, smaller n's/deviations from Ho
- CUSUM more powerful for larger λ's, larger n's/deviations from Ho
- Many Inconclusive Results

Further Study

- Granularity of Data
- Other g's, α's
- Other distributions
- Limitations of TEXAS α vs. CUSUM ARL