FAILURE MODE REMOVAL AS A SERVICE PROCESS

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PROBLEM SITUATION

- SYSTEM
  - New or Upgraded
  - K, e.g. 5, Major Subsystems (k=1,2,..,K)
- FAILURE PRONE
  - FAULTS or FAILURE MODES (FM)
  - FM ACTIVATE RANDOMLY
    - Influenced by Environment (Heat, Precip.)
    - Cascades
- TESTS: TO MITIGATE FMs
  - Developmental (DT): Subsystems, Prototype
  - Operational (OT): Assembled System in Field Environment

Failure Modes (FMs) Introduced During System Design Phase

- FM Definition: Individual Source/Origin of Malfunction/Failure
- Discovered, Removed (or Mitigated) During
  - Developmental Testing (DT); Length D
  - Operational Testing (OT)
    - OT: Field-like conditions
    - More costly than DT
      - Test Design ("Design of Experiments"), Execution, Analysis
        - » Response Metric Important!
      - System Redesign
- Reliability Growth
  - Duane/Crow
  - Hall & Ellner; Hall, Ellner & Mosleh
- Purpose: Field effective and suitable systems

Model for FM Activation Time

- **T**=time until a FM activation
- Model: $P\{\mathbf{T}>t|\mu\}=\exp\{-\mu\theta t\}$
- $P\{\mathbf{T}>t\}=E[P\{\mathbf{T}>t|\mu\}]=E[\exp\{-\theta t\mu\}]$
  - Laplace Transform: $\varphi(\theta t)=E[\exp(-\mu\theta t)]$
- Distribution of μ
  - Gamma: (Transform): $\varphi(\theta t) =(1+(\theta t/\beta))^{-\alpha}$
    $= \exp\{-\alpha \ln(1+(\theta t/\beta))\}$
  - Stable: (Transform): $\varphi(\theta t) =\exp\{-(\theta t/\beta)^{\alpha}\}$
- Mixture (NOT Lévy Process!)

COVARIATES/EXPLANATORY VARIABLES

- For DT: $\theta_D\left(\underline{x}\right)$, $\underline{\text{x}}$=Cov. Vector

  E.G. $\theta_D\left(\underline{x}\right)=c_0+c_1 x_1+...+c_m x_m$

  – Abbreviated θ_D
- For OT: $\theta_O\left(\underline{y}\right)$

  E.G. $\theta_O\left(\underline{y}\right)$ Linear
- Acknowledged, not estimated here

Activation Time: **T**
P{**T**>D+s|**T**>D}

- $\varphi(\theta_D D+\theta_O s)/\varphi(\theta_D D)$
  - Gamma
  - Stable
- Closed-form expressions: quantiles: OT time until activation of FM given FM has not activated during DT: P{**T**>D+s|**T**>D}
- Modeled as IID given G or S, or …

DT: θ_D OT: θ_O

$\theta_D=\theta_O=1$

Time in DT=D=1000;

Shape Parameter=0.5

Probability FM Survives DT	Distribution	Quantile of Time Until Activation in OT Given Does NOT Active in DT			
		0.25	0.5	0.75	0.80
0.25	**Gamma**	830	3200	16000	25600
	Stable	458	1250	3000	3670
0.5	**Gamma**	1037	4000	20000	32000
	Stable	1002	3000	8000	10035
0.75	**Gamma**	1778	6857	34286	54857
	Stable	3000	10624	32859	42487

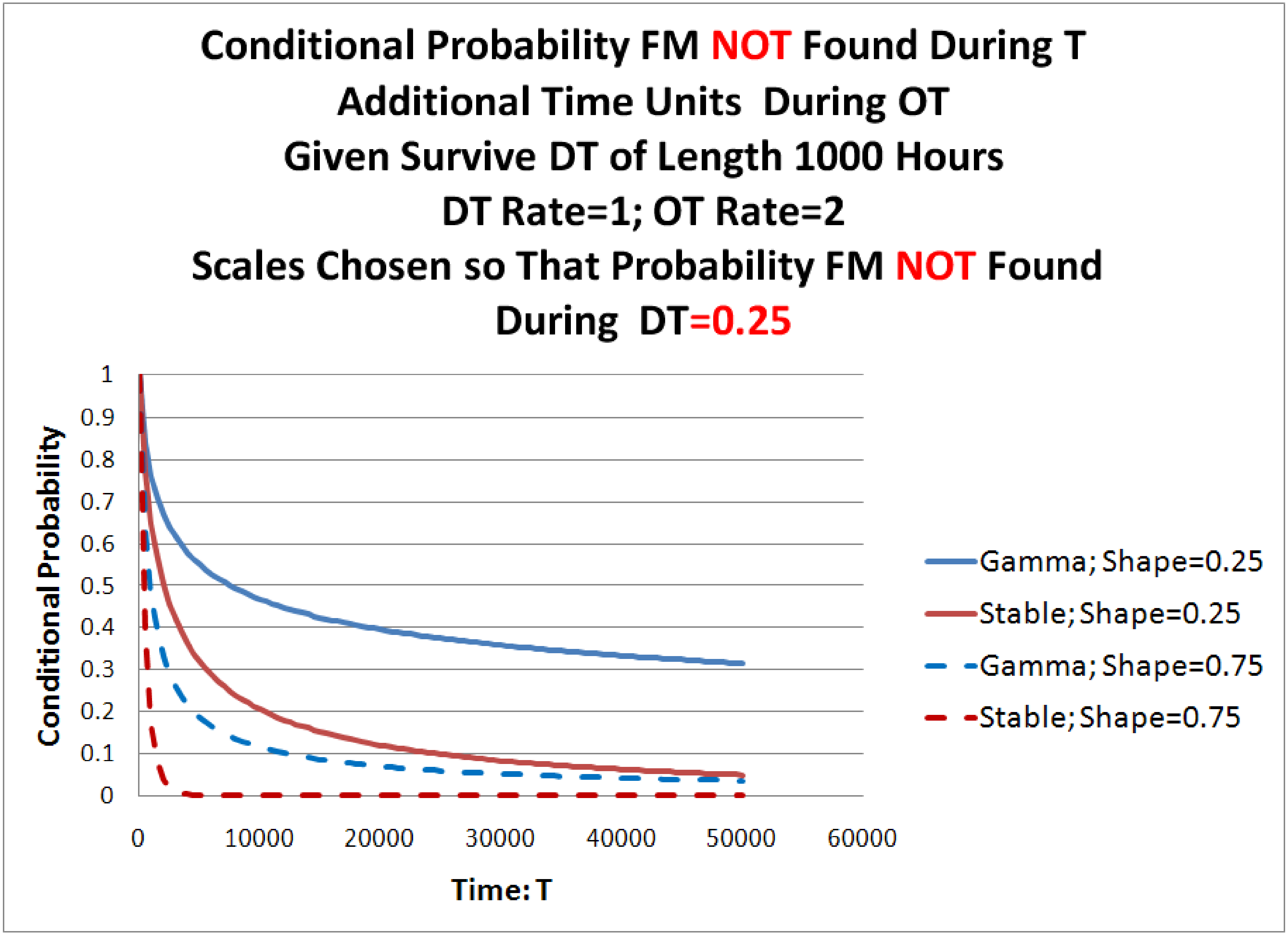
DT : θ_D OT: θ_O

θ_D=1; θ_O=2

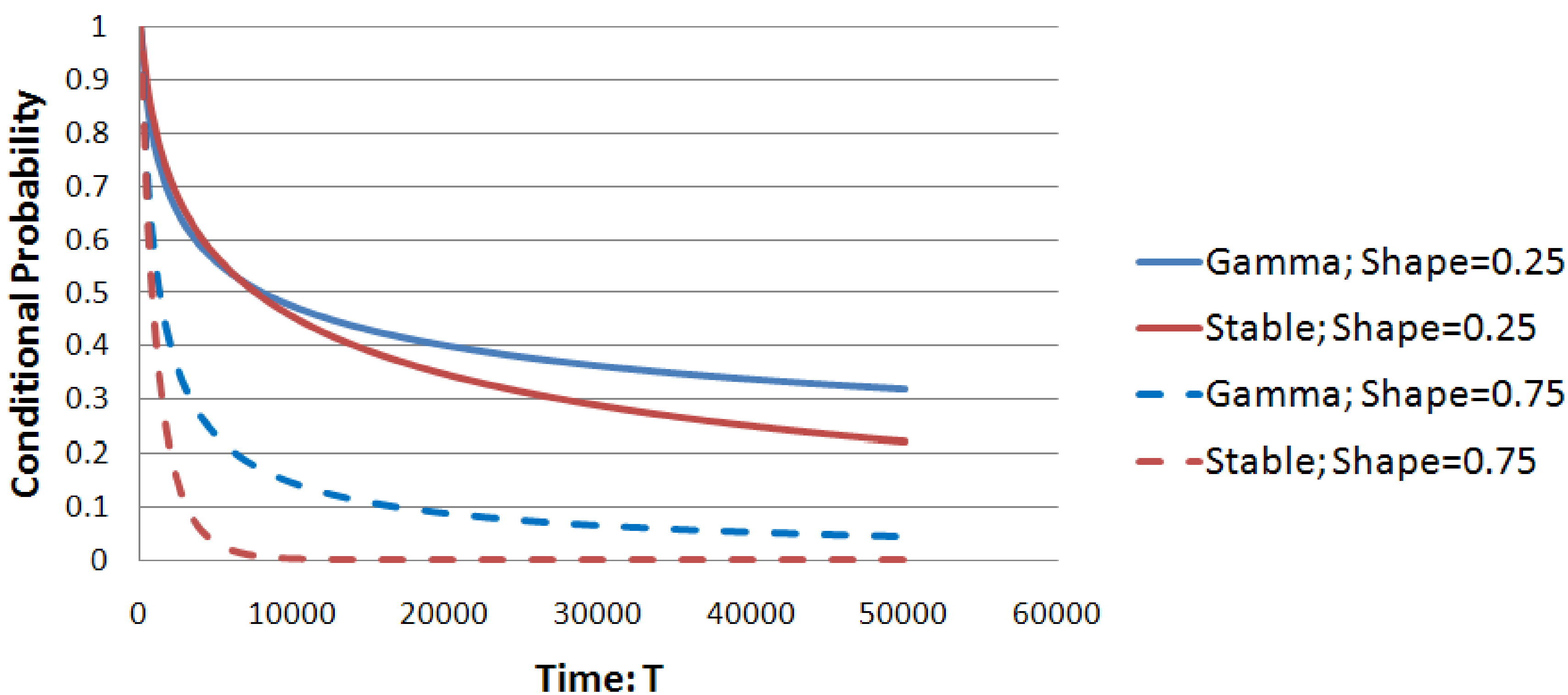
Time in DT=D=1000;

Shape Parameter=0.5

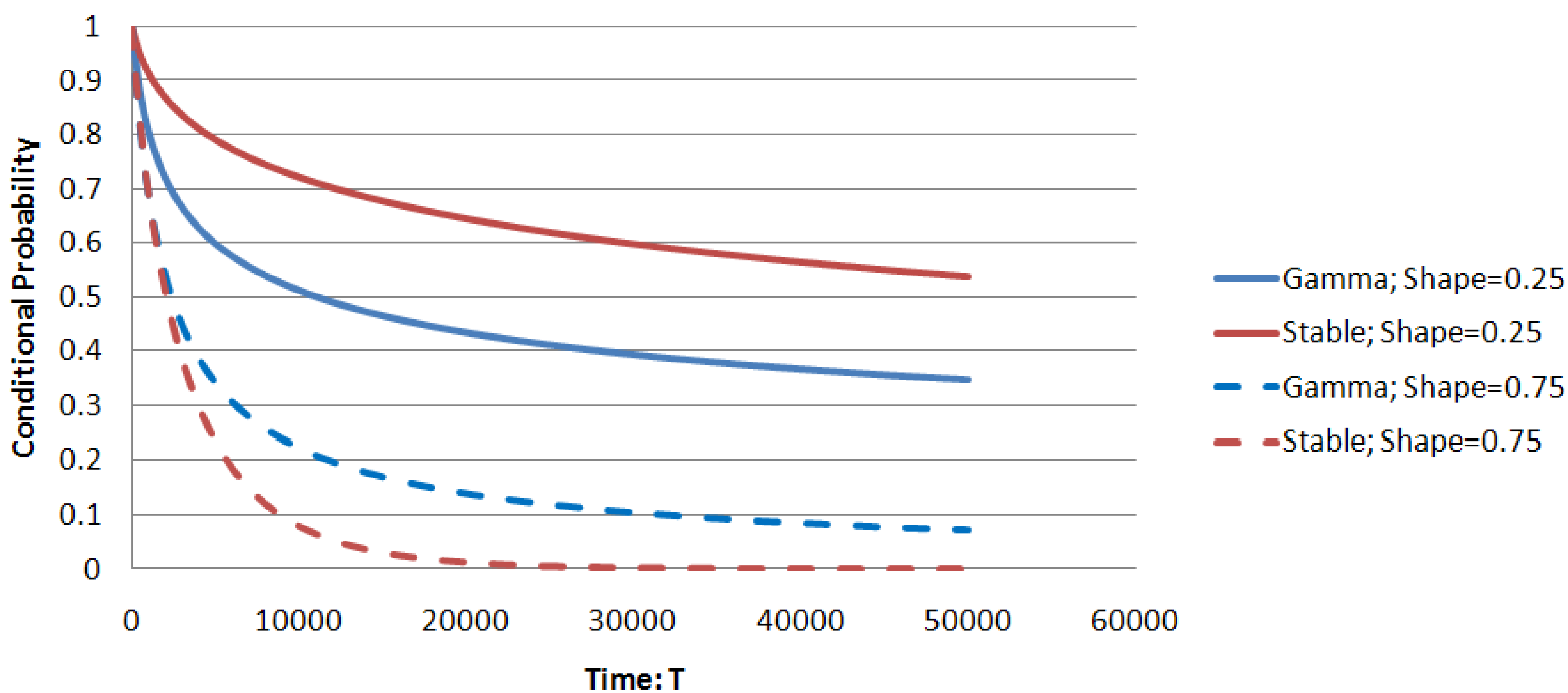
Probability FM Survives DT	Distribution	Quantile of Time Until Activation in OT Given Does NOT Active in DT			
		0.25	0.5	0.75	0.80
0.25	**Gamma**	415	1600	8000	12800
	Stable	229	625	1500	1835
0.5	**Gamma**	519	2000	10000	16000
	Stable	501	1500	4000	5018
0.75	**Gamma**	889	3429	17143	27429
	Stable	1500	5312	16429	21244


Conditional Probability FM NOT Found During T
Additional Time Units During OT
Given Survive DT of Length 1000 Hours
DT Rate=1; OT Rate=2
Scales Chosen so That Probability FM NOT Found
During DT=0.25
Conditional Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10000
20000
30000
40000
50000
60000
Time: T
Gamma; Shape=0.25
Stable; Shape=0.25
Gamma; Shape=0.75
Stable; Shape=0.75

Conditional Probability FM NOT Found During T Additional Time Units During OT Given Survive DT of length 1000 Hours DT Rate=1; OT Rate=2 Scales Chosen so That Probability FM NOT Found During DT=0.5



Conditional Probability FM NOT Found During T Additional Time Units During OT Given Survive DT of Length 1000 Hours DT Rate=1; OT Rate=2 Scales Chosen so That Probability FM NOT Found During DT=0.75



Numbers of FMs

- $R_k(0)$=Random number FMs type k before testing~Poisson **Mean M_k**
- FMs of type k: IID random, mean time until activation:$1/\mu_k$
- Each activation, FM removed, with probability $\rho_k \leq 1$, independent of past other events.
- $R_k(t)$=number of FMs of type k remaining in the system after t time units of testing
  - Not activated yet
  - Activated, but not removed
- Subsequently $\rho_k=\rho_{kD}=0$. Censoring Effect of $\rho_k>0$ Recognized but Not in Current Analysis

Observe Number FMs that Activate in (0,D]

- Number FMs type k activate during DT: $m_k\left(D\right)$
- Initially Poisson number type k FMs, mean M_k
- Probability type k FM activates during D time units=1-$\varphi\left(\theta_{k,D}\, D\right)$

- Estimated mean number of FMs **NOT YET** activated during D time units:

$$\underbrace{m_k\left(D\right)}_{\text{Observed}}\ \underbrace{\varphi\left(\theta_{k,D}\, D\right)/\left[1-\varphi\left(\theta_{k,D}\, D\right)\right]}_{\text{Estimated Based on Observations in DT}}$$

DATA FROM DT

- Times between FM_k Activation:

  ~ ith FM_k: $t_{ik}(1),\ t_{ik}(2),\ \ldots,\ t_{ik}(m_{ik})$

  m_k is Number k-Types Observed

  ~ DT Data Analysis Recommended

- Model:

  $\boldsymbol{T}_{ik}$=Conditional Survival Time

  $$P\left\{\boldsymbol{T}_{ik} > t \mid \boldsymbol{\mu}_{ik} = \mu_{ik}\right\} = \exp\left\{-\mu_{ik}\theta_D t\right\},$$

  $$\text{P}\left\{\boldsymbol{N}_{ik}(t) = n_{ik} \mid \boldsymbol{\mu}_{ik} = \mu_{ik}\right\} = e^{-\mu_{ik}\theta_D t}\left[\frac{\left(\mu_{ik}\theta_D t\right)^{n_{ik}}}{n_{ik}!}\right]$$

  $\boldsymbol{N}_{ik}(t)$ is ***Conditionally*** Poisson $\left(\mu_{ik}\theta_D t\right)$

Estimate Prevailing μ_{ik} (Conditional Value $\boldsymbol{\mu}_{ik}$)

- $MLE: \hat{\mu}_{ik}\left(M\right)=\dfrac{n_{ik}}{\theta_D \min\left(t_{ik}^{*},D\right)}$

where t_{ik}^{*} is the time of removal (if accomplished before D);
n_{ik} is number of activations during DT

- Bayes, Jefferys' (δ) NonInformative Prior:

$$\hat{\mu}_{ik}\left(B\right)=\frac{n_{ik}+1-\delta}{\theta_D \min\left(t_{ik}^{*},D\right)},\ \left(\delta=1,0.5\right)$$

- ALLOWS BETWEEN-FM$_k$'s RATE ASSESSMENT

~ ***Upward Biased***: No account of FM$_k$'s with "small" rates Surviving D

ESTIMATION OF MIXING DISTRIBUTION

- Observations During DT:

$$\mathrm{dH}_k^{+}\left(z,\underline{p}\right)=\frac{\left(1-e^{-z\theta_D D}\right)dH_k\left(z,\underline{p}\right)}{\int_0^{\infty}\left(1-e^{-z\theta_D D}\right)dH_k\left(z,\underline{p}\right)};\quad \underline{p}\text{:Parameters (e.g. scale, shape)}$$

- Transform:

$$\varphi_k^{+}\left(s;\underline{p}\right)=\frac{\varphi_k\left(s;\underline{p}\right)-\varphi_k\left(s+\theta_D D;\underline{p}\right)}{1-\varphi_k\left(\theta_D D;\underline{p}\right)}$$

- If $\exists$ mean ($<\infty$)

$$E\left[\boldsymbol{\mu}|\boldsymbol{T}<D\right]=-\left[\frac{\varphi_k'\left(s;p\right)-\varphi_k'\left(s+\theta_D D;\underline{p}\right)}{1-\varphi_k\left(\theta_D D;\underline{p}\right)}\Big|_{s=0}\right]$$

–E.G. Gamma (scale β, shape α)

$$E\left[\boldsymbol{\mu}|\boldsymbol{T}<D\right]=\left(\frac{\alpha}{\beta}\right)\left[\frac{1-\left(\frac{\beta}{\beta+\theta_D D}\right)^{\alpha+1}}{1-\left(\frac{\beta}{\beta+\theta_D D}\right)^{\alpha}}\right]>\frac{\alpha}{\beta}$$

RATE ESTS. AS OBSERVATIONS ON MIXING DISTRIBUTION

- Simple Moment Ests (Gamma):

$$\sim \frac{1}{m_k}\sum_{i=1}^{m_k}\hat{\mu}_{ik} = E\left[\boldsymbol{\mu}|\boldsymbol{T}<D\right] = \frac{\alpha}{\beta}\left[\frac{1-\left(\frac{\beta}{\beta+\theta_D D}\right)^{\alpha+1}}{1-\left(\frac{\beta}{\beta+\theta_D D}\right)^{\alpha}}\right]$$

$$\sim \frac{1}{m_k}\sum_{i=1}^{m_k}\left(\hat{\mu}_{ik}\right)^h = E\left[\boldsymbol{\mu}^h|\boldsymbol{T}<D\right]$$

$\sim$ Solve for β, α, etc.$\Rightarrow \hat{\beta},\hat{\alpha}$

- Apply Bayes Via Gibbs Sampling

SPECIAL CASE:
$G_k(z; \beta, \alpha)$=Exp$(z; \beta, \alpha=1)$

- $dH_k^+\left(z;\beta\right)=\dfrac{\left(1-e^{-zD}\right)\beta e^{-\beta z}}{\int_0^{\infty}\left(1-e^{-zD}\right)\beta e^{-\beta z}dz}=\dfrac{\left(\beta+D\right)}{D}\left(1-e^{-zD}\right)\beta e^{-\beta z}$
- *Let* m_k = # observed FMs during DT (Time D)

  $r_i=\frac{n_i}{D}$, given $n_i \geq 1$ (# activations) ; $\bar{r}=\frac{1}{m_k}\sum_{i=1}^{m_k} r_i$
- *Log-likelihood*

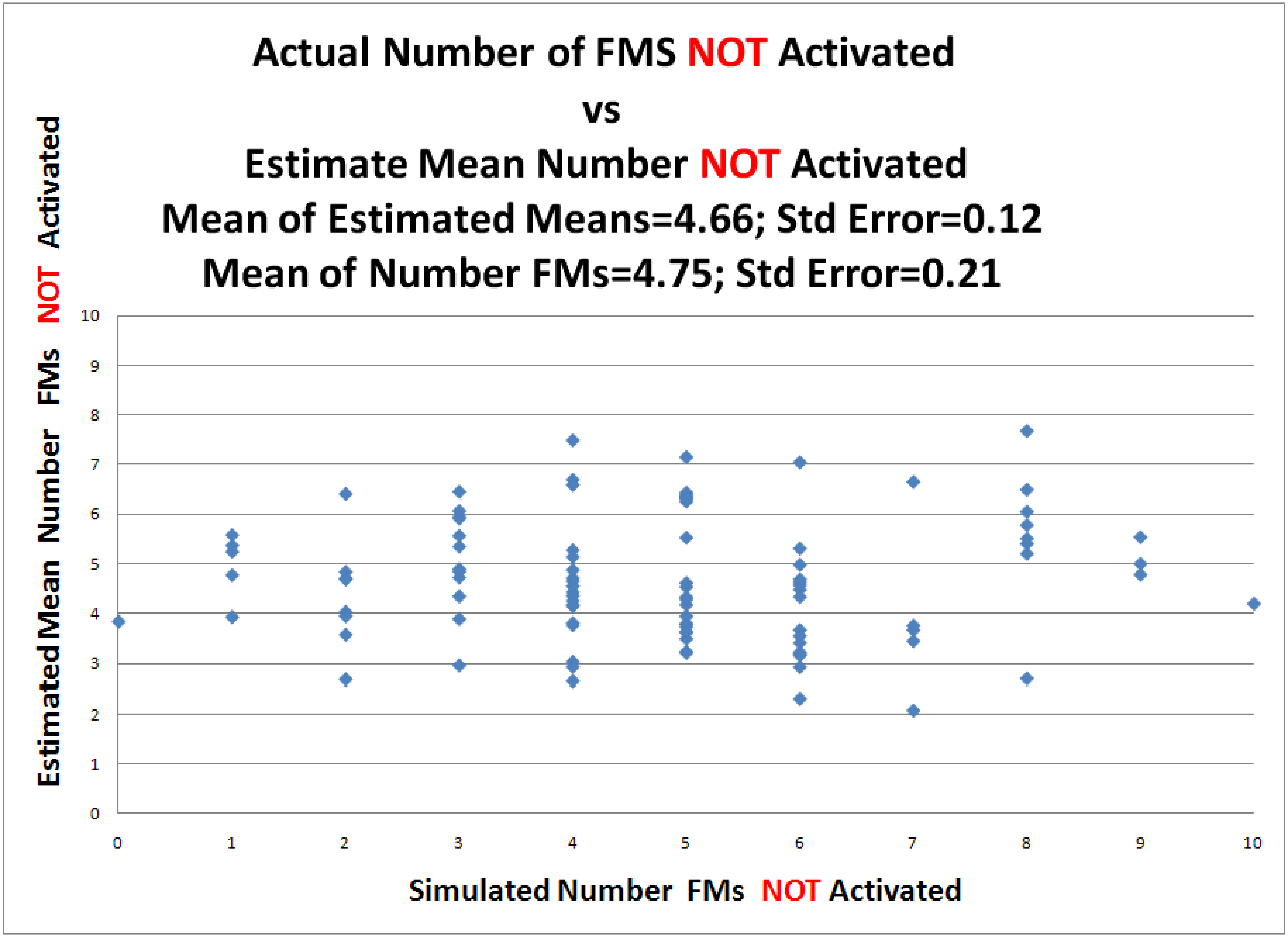
  $\ell\left(\beta;obs.\right)=m_k \ln\left(\beta+D\right)+m_k \ln\left(\beta\right)-\sum_{i=1}^{m_k} r_i$
- *Putting* $\bar{\ell}\left(\beta\right)=\dfrac{\ell\left(\beta;obs.\right)}{m_k}$,

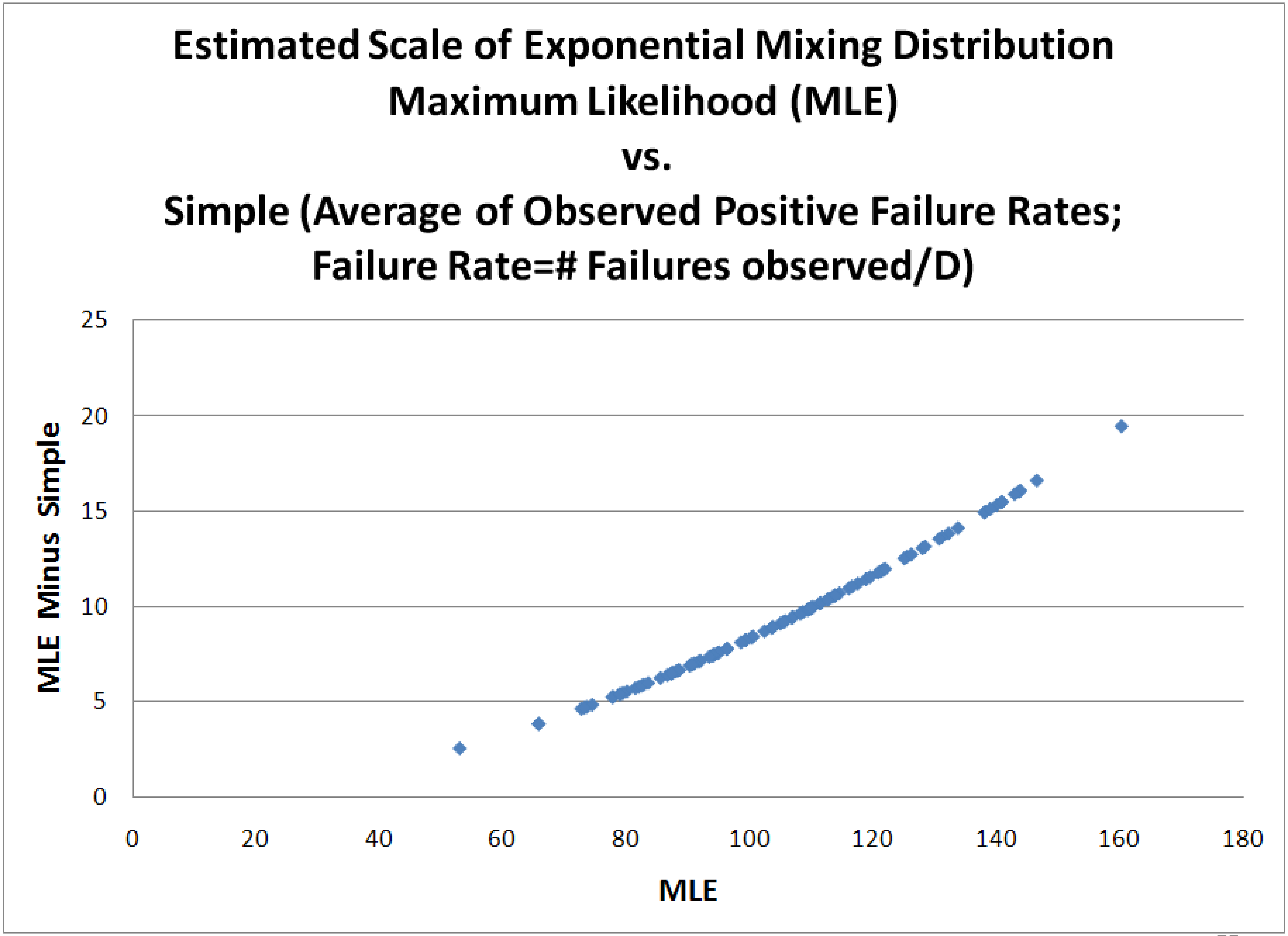
  $\dfrac{\partial\bar{\ell}\left(\beta\right)}{\partial\beta}=\dfrac{1}{\beta+D}+\dfrac{1}{\beta}-\bar{r}=0 \Rightarrow \beta^2\bar{r}+\beta\left[D\bar{r}-2\right]-D=0$

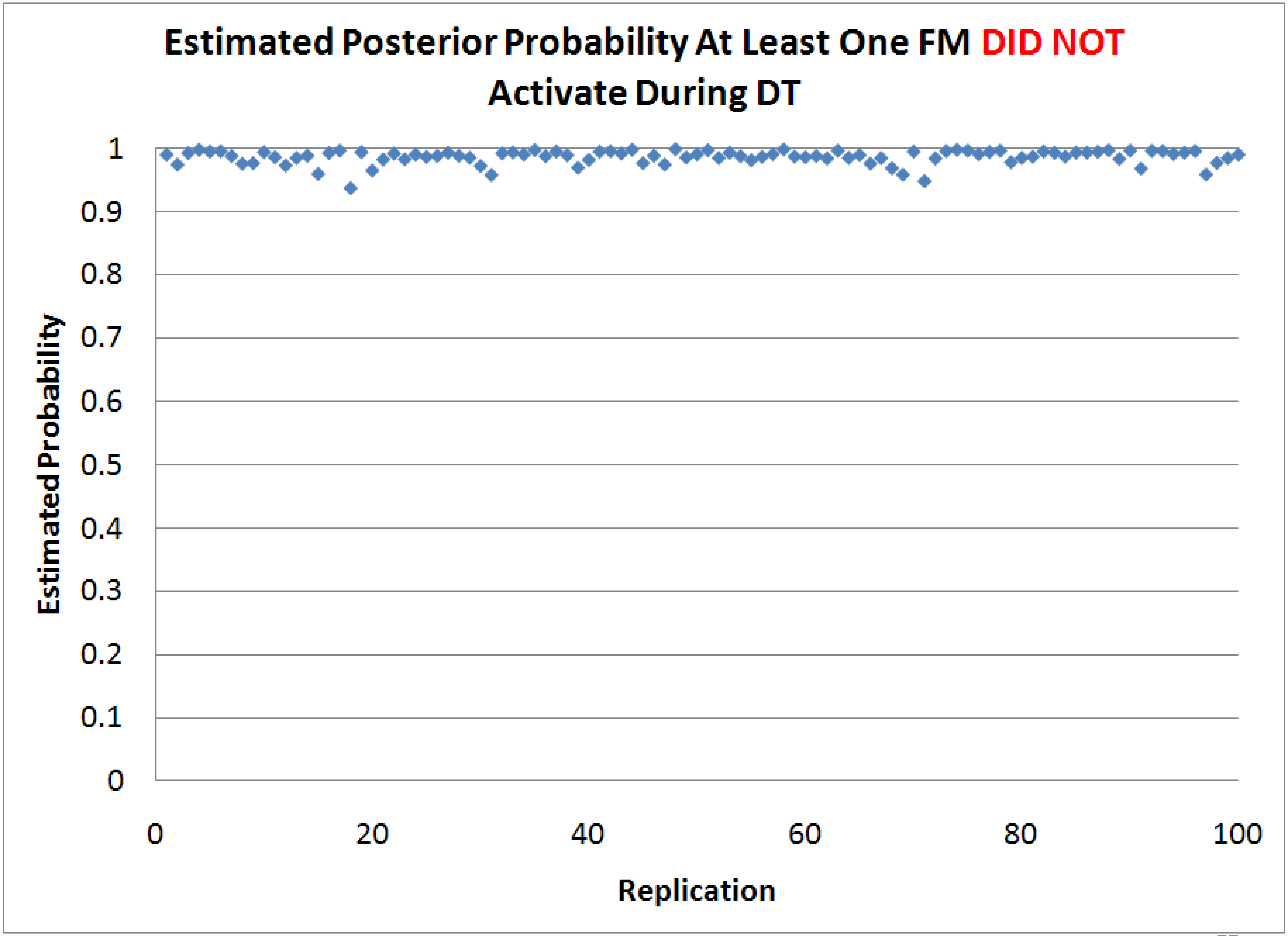
  Result : $\hat{\beta}=\dfrac{-\left(D\bar{r}-2\right)+\sqrt{\left(D\bar{r}-2\right)^2+4\bar{r}D}}{2\bar{r}}$

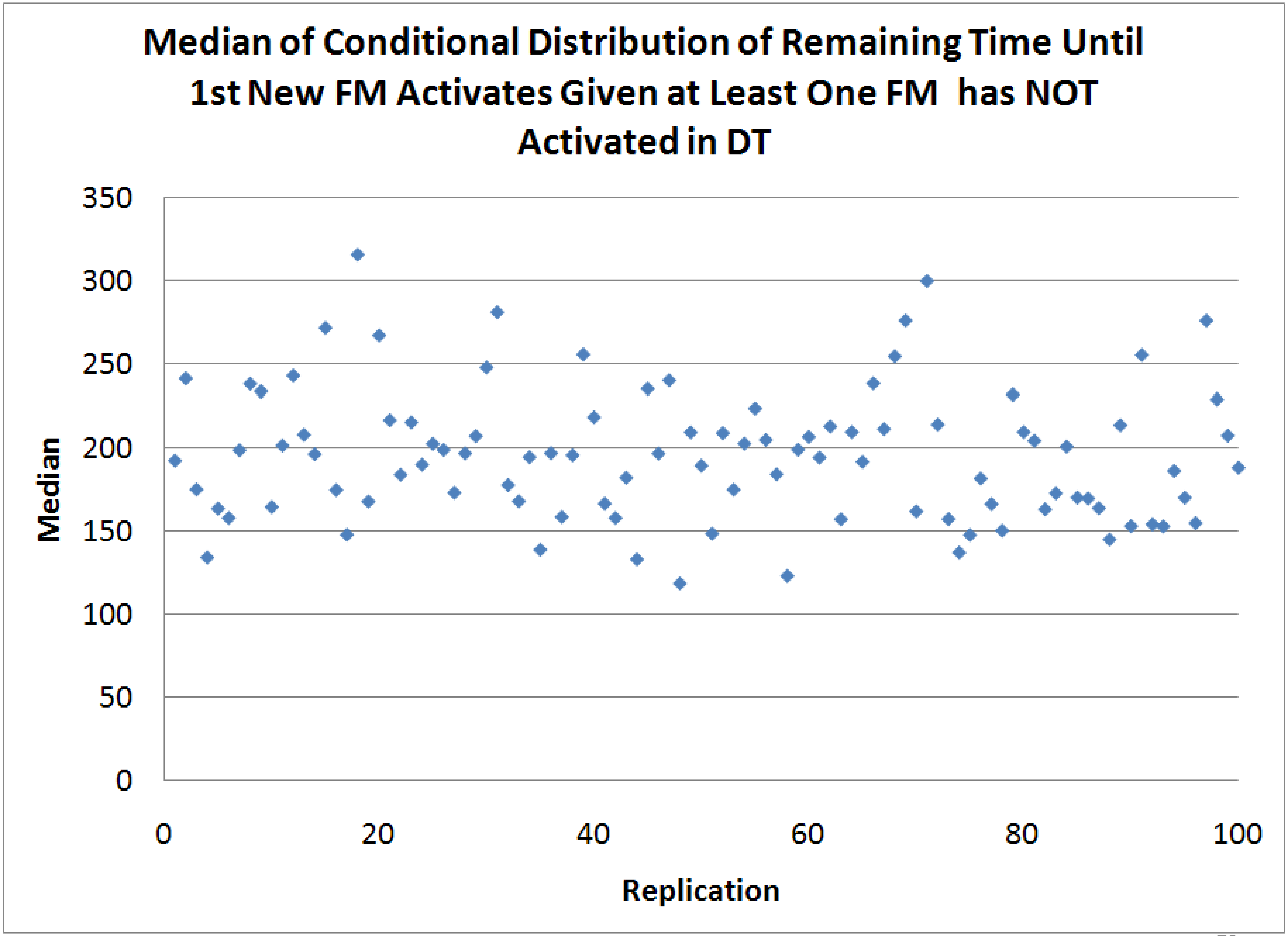
Simulation

- Number of FMs ~ Poisson mean M_k=50
- Mixing Distribution ~ Exponential mean 100
- DT Time=D=1000
- Activated FMs not removed during DT
- 100 Replications


Actual Number of FMS NOT Activated
vs
Estimate Mean Number NOT Activated
Mean of Estimated Means=4.66; Std Error=0.12
Mean of Number FMs=4.75; Std Error=0.21
Estimated Mean Number FMs NOT Activated
10
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10
Simulated Number FMs NOT Activated


Estimated Scale of Exponential Mixing Distribution
Maximum Likelihood (MLE)
vs.
Simple (Average of Observed Positive Failure Rates;
Failure Rate=# Failures observed/D)
MLE Minus Simple
MLE
0
5
10
15
20
25
0
20
40
60
80
100
120
140
160
180


Estimated Posterior Probability At Least One FM DID NOT Activate During DT
Estimated Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
Replication


Median of Conditional Distribution of Remaining Time Until 1st New FM Activates Given at Least One FM has NOT Activated in DT
Median
350
300
250
200
150
100
50
0
0
20
40
60
80
100
Replication

SUMMARY of RESULTS

- ESTIMATES OF (DT-ACTIVATED/OBSERVED) FM RATES
- ESTIMATES OF NUMBERS FM REMAINING
- ESTIMATES OF MIXING DISTRIBUTION PARAMETERS

FUTURE, IN-PROGRESS

- REFINE & EXTEND
  - APPLY TO ACTUAL DATA
  - OT: ANALYSIS WITH K INTERACTING SUBSYSTEMS
  - APPLY TO TEST REPLICATION